

Properties of Exponents & Solving Exponential Equations

Exponents – Division Rule:



1. Simplify:

$$\frac{x^5}{x^3} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = \frac{x^2}{1} = x^2$$

2. What happens to the exponents when dividing two terms that have the same base?

the exponents are subtracted. $5 - 3 = 2$.



3. Use the Exponent Division Rule to simplify each.

a. $\frac{y^{15}}{y^8} = y^7$

b. $\frac{4^7}{4^4} = 4^3$
 $= 64$

c. $\frac{z^5}{z^9} = z^{-4}$



4. In example 3c, what do you notice about the resulting exponent? Explain.

It is negative because $5 - 9 = -4$

Negative Exponents:



1. Simplify using the method demonstrated in example 1 above.

$$\frac{x^3}{x^5} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{1}{x^2}$$

2. Simplify using the Exponent – Division Rule.

$$\frac{x^3}{x^5} = x^{-2}$$

3. What do examples 1 and 2 tell us about negative exponents? Explain.

Here we see that $x^{-2} = \frac{1}{x^2}$

So a negative exponent makes a reciprocal fraction.



4. Re-write each expression as a fraction.

a. $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

b. $y^{-4} = \frac{1}{y^4}$

5. Re-write each fraction as a negative exponent.

a. $\frac{1}{25} = 25^{-1}$
or $= \frac{1}{5^2} = 5^{-2}$

b. $\frac{1}{256} = 256^{-1}$
or $= \frac{1}{4^4} = 4^{-4}$

6. Write the exponential function $f(x) = a \cdot b^x$ represented by the table of values.

x	y
0	2 = 2^1
1	1 = 2^0
2	$\frac{1}{2} = 2^{-1}$
3	$\frac{1}{4} = 2^{-2}$

$f(x) = 2^{(x-1)}$

Roots & Radicals:



1. $\sqrt[n]{a}$ is called a radical where n is called the index and a is called the radicand.

\sqrt{a} is a **square root**. $\sqrt{16} = 4$ because $4 \cdot 4 = 16$

$\sqrt[3]{a}$ is a **cube root**. $\sqrt[3]{64} = 4$ because $4 \cdot 4 \cdot 4 = 64$

$\sqrt[4]{a}$ is a **fourth root**. $\sqrt[4]{256} = 4$ because $4 \cdot 4 \cdot 4 \cdot 4 = \underline{256}$

$\sqrt[5]{a}$ is a **fifth root**. $\sqrt[5]{1024} = 4$ because $\underline{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024}$

And so on.



2. Simplify each radical expression.

a. $\sqrt{81} = 9$ because $9 \cdot 9 = 81$

b. $\sqrt[3]{243} = 3$ because $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

c. $\sqrt[3]{-8} = -2$ because $(-2)(-2)(-2) = -8$

d. $\sqrt{-144} = \uparrow$ because $(-12)(-12) = 144$ and $(12)(12) = 144$.
Impossible two identical #'s can't multiply to a negative value.

Fractional Exponents:



1. The radical expression $\sqrt[r]{a^p}$ can be rewritten using a fractional exponent: $a^{\frac{p}{r}}$.

$$\sqrt[r]{a^p} = a^{\frac{p}{r}}$$

root is in the bottom... just like a tree!

Where p is the POWER and r is the ROOT.



2. Identify the power and the root and then rewrite the expression in radical form.

a. $x^{\frac{3}{2}}$ Power = 3, Root = 2, Radical Form = $\sqrt{x^3}$

b. $10^{\frac{4}{5}}$ Power = 4, Root = 5, Radical Form = $\sqrt[5]{10^4}$

3. Identify the power and the root and then rewrite the expression using a fractional exponent.

a. $\sqrt[4]{x^7}$ Power = 7, Root = 4, Exponent Form = $x^{\frac{7}{4}}$

b. $\sqrt{8^5}$ Power = 5, Root = 2, Exponent Form = $8^{\frac{5}{2}}$

Solving Exponential Equations:



We can sometimes use a common base to solve an exponential equation.

1. Perry is repeatedly folding a piece of paper in half. The number of layers created can be determined by the exponential function $f(x) = 2^x$, where x is the number of folds made.

a. How many layers are created if Perry makes 2 folds? $f(2) = 2^2 = 4$ layers.

b. How many folds must Perry make to create 32 layers?

$$32 = 2^x$$

$$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$$

$$2^5 = 2^x$$

Same base

this means...

$$x = 5$$



2. Use a common base to solve each exponential equation:

a. $3^x = 81$

$$3^x = 3^4$$

$$x = 4$$

$$81 = 3^4$$

b. $4^{x+1} = 1024$

$$4^{x+1} = 4^5$$

$$x+1 = 5$$

$$x = 4$$

$$1024 = 4^5$$

c. $6^{3x} = \frac{1}{36}$

$$\frac{1}{36} = \frac{1}{6^2} = 6^{-2}$$

$$6^{3x} = 6^{-2}$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

3. Write the exponential function $f(x) = a \cdot b^x$ represented by the table of values.

x	y
0	3
3	$\frac{1}{9} = 3^{-2}$
6	$\frac{1}{243}$
9	$\frac{1}{6561}$

y-int = (0, 3) this makes $a = 3$.

$$f(x) = 3b^x$$

$$\frac{1}{9} = 3(b)^3$$

$$\frac{3^{-2}}{3} = \frac{3(b)^3}{3}$$

$$3^{-3} = b^3$$

Negative exponent makes a reciprocal.

$$\left(\frac{1}{3}\right)^3 = b^3$$

$$\frac{1}{3} = b$$

Same power